Graphs are a simple, yet powerful data structure to model complex relationships between real-world data objects. Thus, graph analytics plays an important role in research and industry. As graphs often reflect heterogeneous domain data, their representation requires an expressive data model including the abstraction of graph collections, for example, to analyze communities inside a social network. Further on, answering complex analytical questions about such graphs entails combining multiple analytical operations. To satisfy these requirements, we propose the Extended Property Graph Model, which is semantically rich, schema-free and supports multiple distinct graphs. Based on this representation, it provides declarative and combinable operators to analyze both single graphs and graph collections. Our current implementation is based on the distributed dataflow framework Apache Flink. We present the results of a first experimental study showing the scalability of our implementation on social network data with up to 11 billion edges.

Graph data models are a prerequisite for the execution of graph algorithms, for example, to rank web sites or to analyze social networks. However, complex analytical problems often cannot be solved by a single algorithm as they require a composition of different techniques. Let us give an example: An analyst is interested in the maximum common subgraph among the largest communities inside a social network. To answer this analytical question, we need to identify communities using a specific community detection algorithm, aggregate the number of vertices for each community, select communities above a minimum vertex count and finally apply a dedicated algorithm to find the maximum common subgraph. Furthermore, real-world graphs are usually heterogeneous in terms of the objects they represent and their attached data. For example, vertices of a social network may represent users, groups or bands while relationships may express friendships, memberships or interests. However, even entities of the same type may have heterogeneous attributes, for example, different users may provide more or less information about themselves.

To manage such data and to answer analytical questions like sketched in our example, we identified the following data model requirements: First, a data model must be able to represent single graphs (e.g., the social network) as well as graph collections (e.g., identified communities). Second, it needs to support heterogeneous attributes without a fixed schema not only for vertices and edges but also for graphs (e.g., vertex count). Third, the data model should provide general-purpose operators (e.g., selection by vertex count) as well as support for use-case specific algorithms (e.g., community detection). Fourth, it shall allow the combination of multiple operators and algorithms to analytical programs.

Since these requirements are not met by previous graph data models, we propose the Extended Property Graph Model (EPGM). The EPGM supports not only single but also collections of heterogeneous graphs and includes a wide range of combinable analytical operators. These operators fulfill the closure property as they take single graphs or graph collections as input and result in single graphs or graph collections. The EPGM is implemented as part of GRADOOOP [12], a new system for scalable graph analytics on top of the Hadoop ecosystem. GRADOOOP is GPLv3-licensed and publicly available\(^1\). To the best of our knowledge, this is the first expressive graph data model including native support for graph collections and respective operators implemented on a distributed computing system. Our main contributions can be summarized as follows:

\(^1\)http://www.gradoop.com/
We propose the EPGM, a graph data model that supports not only single graphs but also graph collections with heterogeneous vertices and edges. Our model includes declarative operators for graph analytics.

We describe the first implementation of the EPGM on top of Apache Flink\(^2\), a state-of-the-art distributed dataflow framework.

We present first experimental results to show the scalability of our implementation by applying an analytical program to a social network with up to 11 billion edges.

The remainder of this article is organized as follows: In section 2, we present the EPGM and its operators. Afterward in section 3, we give a brief introduction to Flink, its programming concepts and how the EPGM is mapped to these concepts. The results of our first experiments are reported in section 4. Finally, we discuss related work in section 5 and conclude our work in section 6.

2. EXTENDED PROPERTY GRAPH MODEL

We introduce a data model extending the popular property graph model\(^1\) by support for graph collections and by combinable analytical operators. Graph collections are a natural way to represent logical partitions of a graph, e.g., communities in a social network\(^7\) or business process executions\[^{14}\]. Further on, graph collections are the result of certain graph algorithms, e.g., embeddings found by graph pattern matching\[^{8}\] or frequent subgraph mining\[^{11}\]. The EPGM supports operators for graphs and graph collections as well as their composition to analytical programs. In the following, we will discuss graph representation and provided operators in more detail.

2.1 Graph Representation

In its basic form, a directed graph \(G = (V, E)\) consists of a set of vertices \(V\) and a set of binary edges \(E \subseteq V \times V\). Several extensions of this basic abstraction have been proposed to define a graph data model\[^{2, 3}\]. One of these models, the property graph model (PGM)\[^{17}\], gained wide acceptance and is used in many graph database systems (e.g., Neo4j\(^3\) or Titan\(^4\)). A property graph is a directed, labeled and attributed multigraph. To express heterogeneity, type labels can be defined for vertices and edges (e.g., Person or likes). Attributes have the form of key-value pairs (e.g., name:Alice or age:42) and are referred to as properties. Such properties are set at the instance level without an upfront schema definition. In contrast to the directed and labeled graph model RDF\(^5\), attributes are encapsulated in vertices and edges.

With regard to the requirements stated in our introduction, the PGM is missing support for graph collections and associated operators. To meet all requirements, we have developed the Extended Property Graph Model (EPGM). In this model, a database consists of multiple property graphs which we call logical graphs. These graphs are application-specific subsets from shared sets of vertices and edges, i.e., may have common vertices and edges. Additionally, not only vertices and edges but also logical graphs have a type label and can have different properties. Formally, we define the EPGM database as follows:

\[ \text{EPGM database} DB = (V, E, \mathcal{L}, T, \tau, K, A, \kappa) \text{ consists of a vertex set } V = \{v_i\}, \text{ an edge set } E = \{e_k\} \text{ and a set of logical graphs } \mathcal{L} = \{G_m\}. \text{ Vertices, edges and (logical) graphs are identified by the respective indices } i, k, m \in \mathbb{N}. \text{ An edge } e_k = (v_i, v_j) \text{ with } v_i, v_j \in V \text{ directs from } v_i \text{ to } v_j \text{ and supports loops (i.e., } i = j). \text{ There can be multiple edges between two vertices differentiated by distinct identifiers. A logical graph } G_m = (V_m, E_m) \text{ is an ordered pair of a subset of vertices } V_m \subseteq V \text{ and a subset of edges } E_m \subseteq E \text{ where } \forall (v_i, v_j) \in E_m: v_i, v_j \in V_m. \text{ Logical graphs may potentially overlap such that } V_G \subseteq V, G \subseteq \mathcal{L} = \{V(G_1) \cap V(G_2) \} \geq 0 \land |E(G_1) \cap E(G_2)| \geq 0. \text{ For the definition of type labels we use label alphabet } T \text{ and a mapping } \tau : (V \cup E \cup \mathcal{L}) \rightarrow T. \text{ Similarly, properties (key-value pairs) are defined by key set } K, \text{ value set } A \text{ and mapping } \kappa : (V \cup E \cup \mathcal{L}) \rightarrow (V \cup E \cup \mathcal{L}) \times K \rightarrow A. \]

Figure 1 shows an example EPGM database \(DB\) of a simple social network. Formally, \(DB\) consists of the vertex set \(V = \{v_0, ..., v_9\}\) and the edge set \(E = \{e_0, ..., e_19\}\). Vertices represent persons, forums and interest tags, denoted by corresponding type labels (e.g., Person) and are further described by their properties (e.g., name:Alice). Edges describe the relationships between vertices and also have type labels (e.g., knows) and properties. The key set \(K\) contains all property keys, for example, name, city and since, while the value set \(A\) contains all property values, for example, Alice, Leipzig and 2015. Vertices with the same type label may have different property keys, e.g., \(v_0\) and \(v_1\).

The sample database contains the set of logical graphs \(\mathcal{L} = \{G_0, G_1, G_2\}\), where each graph represents a community inside the social network, in particular specific interest groups (e.g., Databases). Each logical graph has a dedicated subset of vertices and edges, for example, \(V(G_0) = \{v_0, v_1\}\) and \(E(G_0) = \{e_0, e_1\}\). Considering \(G_0\) and \(G_2\), one can see that vertex and edge sets may overlap since \(V(G_0) \cap V(G_2) = \{v_0, v_1\}\) and \(E(G_0) \cap E(G_2) = \{e_0, e_1\}\). Note that also logical graphs have type labels (e.g., Community) and may have properties, which can be used to describe the graph by annotating it with specific metrics (e.g., vertexCount:3) or general information about that graph (e.g., interest:Databases).

Logical graphs, such as those of our example, are either declared explicitly or output of a graph algorithm, e.g., community detection or graph pattern matching. In both cases, they can be used as input for subsequent operators.

2.2 Operators

The EPGM provides operators for single logical graphs and graph collections; operators may also return single logical graphs or graph collections. Here, a graph collection \(G \in \mathcal{L}\) is a n-tuple of logical graphs and may contain duplicate elements. Collections are ordered to support application-specific sorting and position-based selection of logical graphs. In the following, we use the terms collection and graph collection as well as graph and logical graph interchangeably. Table 1 lists our analytical operators together with their corresponding pseudocode syntax for calling them in our domain specific language GrALa (Graph Analytical Language). The syntax adopts the concept of higher-order functions for several operators (e.g., to use aggregate or predicate functions as operator arguments). Based on the input of operators, we distinguish between graph operators and collection operators as well as unary and binary operators (single graph/collection vs. two graphs/collections as input). There are

\(^2\)http://link.apache.org/
\(^3\)http://www.neo4j.com/
\(^4\)http://thinklukeio.github.io/titan/
\(^5\)http://www.w3.org/RDF/
also auxiliary operators to apply graph operators on collections or to call specific graph algorithms. In addition to the listed ones we provide operators to create graphs, vertices and edges including respective labels and properties. In the following, we discuss the operators in more detail. Since the scope of this article is the data model and its analytical capabilities, complex operators (e.g., pattern matching and grouping) are only sketched and will be thoroughly described in future publications.

**Aggregation.** An operator often used in analytical applications is aggregation, where a set of values is mapped to a single value of significant meaning. In the EPGM, we support aggregation at the graph level. Formally, the operator maps an input graph \( G \) to an output graph \( G' \) and applies the user-defined aggregate function \( \alpha : L \to A \). Thus, the resulting graph is a modified version of the input graph with an additional property \( k \), such that \( \kappa(G', k) \Rightarrow \alpha(G) \).

In the following, we show a simple vertex count example:

\[
\begin{align*}
\text{alpha} &= (g \Rightarrow g.V.count()) \\
\text{outGraph} &= \text{inGraph.agregate}('\text{vertexCount}', \text{alpha})
\end{align*}
\]

Here, a user-defined aggregate function alpha computes the cardinality of the vertex set of an input graph \( g.V \). The aggregation operator is called on the graph inGraph with property vertexCount and aggregate function alpha as arguments. The resulting logical graph is assigned to the variable outGraph and provides a property vertexCount storing the result of alpha. Basic aggregate functions such as \( \text{count, sum, min and max} \) are predefined in GrALa and can be applied to vertex and edge collections.

**Transformation.** The structure-preserving transform operator allows the in-place modification of graph, vertex and edge data, for example, to reduce data volume for further processing or to align different schemata during data integration. The operator applies the user-defined transformation functions \( \gamma : L \to L, \nu : V \to V \) and \( \epsilon : E \to E \) to an input graph \( G \), and outputs the graph \( G' = \gamma(G) \) where \( V(G') = \{ v \nu | v \in V(G) \} \), \( E(G') = \{ \epsilon(e) | e \in E(G) \} \). The transformation functions are able to modify type labels as well as property keys and values, but preserve identifiers and the graph structure. In the following example, we define the three transformation functions \( \gamma, \nu \) and \( \epsilon \) and use them to transform the graph \( G_0 \) (db.G[0]) of Figure 1:

\[
\begin{align*}
\text{gamma} &= (gIn, gOut => gOut['topic'] = gIn['interest']) \\
\text{nu} &= (vIn, vOut => vOut.label = vIn['name']) \\
\text{epsilon} &= (eIn, eOut => eOut.label = eIn.label)
\end{align*}
\]

\[
\begin{align*}
\text{outGraph} &= \text{db.G[0].transform(gamma, nu, epsilon)}
\end{align*}
\]

The graph transformation function gamma takes the current graph instance gIn and the new graph instance gOut as input. The latter is a copy of the current graph with omitted type label and properties. The function determines that graph label and all graph properties are removed, except graph property interest, which is renamed to topic. The definition of vertex and edge transformations is analogous.

**Pattern Matching.** A fundamental operation of graph analytics is the retrieval of subgraphs isomorphic to a user-defined pattern graph. For example, given a social network, an analyst may be interested in all pairs of users who are members of the same forum with a specific tag. To support such queries, we provide the pattern matching operator, where a pattern graph \( G^* \) and a predicate \( \varphi : L \to \{\text{true, false}\} \) are the operator arguments. Pattern matching is applied to a graph \( G \) and returns a graph collection \( G' = \{ G' | G' \subseteq G \land G' \simeq G^* \land \varphi(G') = \text{true} \} \) containing all matches, for example:

\[
\begin{align*}
\text{outCollection} &= \text{db.G.match}(''
\text{(a:Person)<-[:hasMember]->(b:Forum)}
\text{(c:Person)<-[:hasMember]->(b)}
\text{(b)-[g:hasTag]->(d:Tag \{name = \text{"Databases"}\})}''
\end{align*}
\]

The shown pattern graph reflects our social network query.

For GrALa, we adopted the basic concept of describing graph patterns using ASCII characters from Neo4j Cypher\(^6\), where \((a)-[e]->(b)\) denotes an edge e from vertex a to vertex b. The predicate function \( \varphi \) is embedded into the pattern by defining type labels and properties. In the example, we describe a pattern of four vertices and three edges, which are assigned to variables \( a, b, c, d \) for vertices; \( e, f, g \) for edges). Variables are optionally followed by a label (e.g., \( a:Person \)) and properties (e.g., \( \{\text{name = \text{"Databases"}}\} \)). The operator is called for the logical graph representing the whole

\(^6\)http://neo4j.com/docs/2.3.1/cypher-query-lang.html
database $DB$ of Figure 1 and returns a collection assigned to variable `outCollection` and containing a single logical graph: $G' = \{(\{v_0, v_1, v_2, v_3\}, \{v_{10}, e_{11}, e_{16}\})\}$. An EBNF grammar of the pattern definition language is already publically available\(^7\) and used by Gradoop.

**Subgraph.** If a specific subgraph is of interest, we provide an operator to extract that subgraph by applying user-defined predicate functions $\varphi_v : V \rightarrow \{\text{true, false}\}$ and $\varphi_e : E \rightarrow \{\text{true, false}\}$ which results in a new graph $G' \subseteq G$ with $V(G') = \{v \mid v \in V(G) \land \varphi_v(v) = \text{true}\}$ and $E(G') = \{(v_i, v_j) \mid (v_i, v_j) \in E(G) \land \varphi_e((v_i, v_j)) = \text{true}\}$. Furthermore, $E(G') = \{(e_1', e_2', ..., e_l')\}$ where $e_i'$ is a `super edge` and $s_i(u, v)$ is the super edge of $\langle u, v \rangle$. Edge groups are determined among the super vertices and a set of edge keys $K_e \subseteq K_e$ such that $V'(u, v), (s, t) \in E(G) : s_i(u, v) = s_i(s, t) \Rightarrow s_i(u) = s_i(s) \land s_i(v) = s_i(t) \land \forall k \in K_e : \kappa(s, t, k) = \kappa(s_i(u), s_i(v), k)$. Additionally, the vertex and edge aggregate functions $\gamma_v : \mathcal{P}(V) \rightarrow A$ and $\gamma_e : \mathcal{P}(E) \rightarrow A$ are used to compute aggregated property values for grouped vertices and edges, e.g., the average age of persons in a group or the number of group members. The aggregate value is stored at the super vertex and super edge respectively. The following example shows the application of our grouping operator using GrALa:

```java
outGraph = dbG .subgraph( 
  (v => v.label == 'Person' or v.label == 'Forum'),
  (e => e.label == 'hasModerator'))
```

We use nested predicate functions to filter vertices and edges based on the relevant type labels. Applied to the database graph (dbG), the operator returns a graph described through $G' = \{(v_0, v_3, v_5, v_6)\}$. By omitting either a vertex or an edge predicate function exclusively, the operator is also suitable to declare vertex-induced or edge-induced subgraphs respectively.

**Grouping.** The `groupBy` operator determines a structural grouping of vertices and edges to condense a graph and thus helps to uncover insights about patterns hidden in the graph. Let $G'$ be the grouped graph of $G$, then each vertex in $V(G')$ represents a group of vertices in $V(G)$; edges in $E(G')$ represent a group of edges between the vertex group members in $V(G)$. More formally, $V(G') = \{v'_1, v'_2, ..., v'_l\}$ where $v'_i$ is called a `super vertex` and $\forall v \in V(G)$, $s_v(v)$ is the super vertex of $v$. Vertices are grouped based on their property values, such that for a given set of vertex property keys $K_v \subseteq K_v, \forall u, v \in V(G) : s_v(u) = s_v(v) \Rightarrow \forall k \in K_v : \kappa(u, k) = \kappa(v, k) = \kappa(s_v(u), k)$. Furthermore, $E(G') = \{(e_1', e_2', ..., e_l')\}$ where $e_i'$ is a `super edge` and $s_i(u, v)$ is the super edge of $\langle u, v \rangle$. Edge groups are determined among the super vertices and a set of edge keys $K_e \subseteq K_e$ such that $V'(u, v), (s, t) \in E(G) : s_i(u, v) = s_i(s, t) \Rightarrow s_i(u) = s_i(s) \land s_i(v) = s_i(t) \land \forall k \in K_e : \kappa(s, t, k) = \kappa(s_i(u), s_i(v), k)$. Additionally, the vertex and edge aggregate functions $\gamma_v : \mathcal{P}(V) \rightarrow A$ and $\gamma_e : \mathcal{P}(E) \rightarrow A$ are used to compute aggregated property values for grouped vertices and edges, e.g., the average age of persons in a group or the number of group members. The aggregate value is stored at the super vertex and super edge respectively. The following example shows the application of our grouping operator using GrALa:

```java
outGraph = dbG .subgraph( 
  (v => v.label == 'Person'),
  (e => e.label == 'knows')) .groupBy( 
  [:label, 'city'],
  (superVertex, vertices => superVertex['count'] = vertices.count()),
  [:label],
  (superEdge, edges => superEdge['count'] = edges.count()))
```

The goal of this example is to group persons in the graph of Figure 1 by the city they live in and to calculate the number of group members. Furthermore, we want to group edges between users living in different cities as well as such living in the same city. First, we use the subgraph operator to describe the input graph for grouping consisting of all persons and their mutual relationships. In line 6, we define the vertex grouping keys. Here, we want to group vertices by their type label (denoted by the symbol :label) and property key city. Edges are grouped only by type label (line 9). In lines 7 and 10, we define the vertex and edge aggregate functions. Both receive the super entity (i.e., `superVertex`, `superEdge`) and the set of group members (i.e., `vertices`, `edges`) as input. Both functions apply the aggregate function `count()`

![Table 1: EPGM operators and their implementation state (as of April 2016, wip: work in progress)](https://github.com/s1ck/gdl)

### Table 1: EPGM operators and their implementation state (as of April 2016, wip: work in progress)

<table>
<thead>
<tr>
<th>Operator</th>
<th>GrALa</th>
<th>Operator Signature</th>
<th>Output</th>
<th>Impl. State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregation</td>
<td>Graph.aggregate( propertyKey, aggregateFunction )</td>
<td>Graph</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Transformation</td>
<td>Graph.transform( functionDefinition, vertexFunction, edgeFunction )</td>
<td>Graph</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Subgraph</td>
<td>Graph.subgraph( vertexPredicateFunction, edgePredicateFunction )</td>
<td>Graph</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Grouping</td>
<td>Graph.groupby( vertexGroupingKeys, vertexAggregateFunction, edgeGroupingKeys, edgeAggregateFunction )</td>
<td>Graph</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Collection</td>
<td>Collection.select( predicateFunction )</td>
<td>Collection</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Limit</td>
<td>Collection.limit( n )</td>
<td>Collection</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Sorting</td>
<td>Collection.sort( propertyKey, [:asc;desc] )</td>
<td>Collection</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Equality</td>
<td>Graph.equals( otherGraph, [:identity:datatype] )</td>
<td>Boolean</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Combination</td>
<td>Graph.combine( otherGraph )</td>
<td>Graph</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Exclusion</td>
<td>Graph.exclude( otherGraph )</td>
<td>Graph</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Overlap</td>
<td>Graph.overlap( otherGraph )</td>
<td>Graph</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Difference</td>
<td>Collection.difference( otherCollection )</td>
<td>Collection</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Intersect</td>
<td>Collection.intersect( otherCollection )</td>
<td>Collection</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Union</td>
<td>Collection.union( otherCollection )</td>
<td>Collection</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

\(^7\)https://github.com/s1ck/gdl
Figure 2: Result graph of Grouping example

on the set of grouped entities to compute the group size. The resulting value is stored as a new property count at the super vertex and super edge respectively. Figure 2 shows the resulting logical graph of the grouping example.

**Binary Graph Operators.** The EPGM includes binary graph operators to compare two input graphs. The equality operator determines if two graphs are equal according to a given equality function $\xi: \mathcal{L} \times \mathcal{L} \rightarrow \{\text{true}, \text{false}\}$. We provide two implementations of $\xi$: :identity and :data. The first one determines equality based on vertex and edge identifiers, thus evaluates to true, if both input graphs contain the same instances. The second one compares the input graphs using a canonical form [11] representing labels and properties of the contained vertices and edges. Further binary graph operators are adopted from set-theory and determine the union (combination operator), intersection (overlap) and difference (exclusion) of two graphs resulting in a new graph. We denote these operators by dedicated terms to distinguish them from set-theoretic operators on graph collections (see next paragraph).

For example, the combination operator is useful to merge previously selected subgraphs into a new graph. The combination of input graphs $G_i, G_j$ is a graph $G'$ consisting of the vertex set $V(G') = V(G_i) \cup V(G_j)$ and the edge set $E(G') = E(G_i) \cup E(G_j)$. For our example graph in Figure 1, the call `db.G[0].combine(db.G[2])` results in the new graph $G' = \{v_0, v_1, v_2, v_3\}$. Similarly, the overlap of two graphs $G_i, G_j$ is a graph $G'$ with vertex set $V(G') = V(G_i) \cap V(G_j)$ and edge set $E(G') = E(G_i) \cap E(G_j)$. Applying the exclusion operator to $G_i$ and $G_j$ determines all $G_i$ elements that do not occur in $G_j$, i.e., $V(G') = V(G_i) \setminus V(G_j)$ and $E(G') = \{(u, v) \in E(G_i) \mid u, v \in V(G')\}$.

**Collection Operators.** Collection operators are such requiring a graph collection as input. For example, the selection operator selects those graphs from an input collection $\mathcal{G}$ for which a user-defined predicate function $\varphi: \mathcal{L} \rightarrow \{\text{true}, \text{false}\}$ evaluates to true. The output is a collection $\mathcal{G}'$ containing all qualifying graphs, such that $\mathcal{G}' = \{G \in \mathcal{G} \mid \varphi(G) = \text{true}\}$. Predicates are typically based on aggregated graph properties, as shown in the following example:

```scala
incollection = <db.G[0],db.G[1],db.G[2]>
phi = (g => g['vertexCount'] > 3)
outcollection = incollection.select(phi)
```

Here, the operator is applied to a collection of three logical graphs. The operator argument, a user-defined predicate function phi, will evaluate to true if the input graph has a value greater than 3 for property key vertexCount. Applied to the database of Figure 1, the result collection only contains db.G[2].

As shown in Table 1, we further support unary operators for eliminating duplicate graphs in collections based on their unique identifier (distinct), for selecting $n (n \in \mathbb{N})$ graphs from a collection (limit) and for sorting collections (sort). The sort operator returns a collection sorted by a graph property $k$ in either ascending or descending order. There are also binary operators that can be applied on two collections. Similar to graph equality, collection equality determines, if two collections are equal according to a provided equality function $\xi: \mathcal{L}^n \times \mathcal{L}^n \rightarrow \{\text{true}, \text{false}\}$. Collections are considered to be equal if their elements are equal either with regard to their unique identifiers (:identity) or to the data they hold (:data). Additionally, the set-theoretical operators union, intersection and difference compute new collections based on graph identifiers.

**Auxiliary Operators.** In addition to the presented graph and collection operators, advanced graph analytics often require the use of application-specific graph mining algorithms. One application can be the extraction of subgraphs that cannot be achieved by pattern matching, e.g., the detection of communities in a social network [7]. To support the plug-in of external algorithms, we provide generic call operators, which may have graphs and graph collections as input or output. Depending on the output type, we distinguish between so-called callForGraph (single graph result) and callForCollection operators.

Furthermore, it is often necessary to execute a unary graph operator on more than one graph, for example, to compute an aggregated value for all graphs in a collection. Not only the previously introduced operators aggregation, transformation and grouping, but all other operators with single logical graphs as input and output (i.e., $op: \mathcal{L} \rightarrow \mathcal{L}$) can be executed on each element of a graph collection using the apply operator. Let $\mathcal{G} = \{G_1, G_2, \ldots, G_n\}$ be an input collection, then the output is $G' = \{op(G_1), op(G_2), \ldots, op(G_n)\}$ under preservation of cardinality and order. The following example shows an aggregation which computes the edge count and is applied to all logical graphs in the collection:

```scala
outcollection = incollection.apply(g =>
  g.aggregate('edgeCount', (h => h.E.count())))
```

Similarly, in order to apply a binary operator on a graph collection, we adopt the reduce operator as often found in functional programming languages. The operator takes a graph collection and a commutative binary graph operator as input. The binary operator $op: \mathcal{L} \times \mathcal{L} \rightarrow \mathcal{L}$ is initially applied on the first pair of elements of the input collection which results in a new graph. This result graph and the next element from the input collection are then the new arguments for the binary operator and so on. In this way, the binary operator is applied on pairs of graphs until all elements of the input collection are processed and the final graph is computed. In the following example, we call the reduce operator parametrized with the combine operator on all graphs in the given collection to compute a single graph:

```scala
outGraph = inCollection.reduce(g,h => g.combine(h))
```

3. IMPLEMENTATION

The most recent approaches to large-scale graph analytics are libraries on top of distributed dataflow frameworks, e.g., GraphX on Apache Spark [18] or Gelly on Apache Flink. These libraries are well suited for executing iterative graph algorithms on distributed graphs in combination with general data transformation operators provided by the underlying frameworks. However, the implemented graph data models have no support for collections and are generic,
which means arbitrary user-defined data can be attached to vertices and edges. In consequence, model-specific operators, for example, such based on properties, need to be user-defined, too. Hence, using those libraries to solve complex analytical problems becomes a laborious task.

We implemented the EPGM on top of Apache Flink to provide new features for graph analytics and to benefit from existing capabilities to large-scale data and graph processing at the same time. In this section, we will briefly introduce Flink and its programming concepts. We will further show how the EPGM graph representation and a subset of the introduced operators are mapped to those concepts.

### 3.1 Apache Flink

Apache Flink is the successor of the former research project Stratosphere [1] and supports the declarative definition and distributed execution of analytical programs on data flows sourced from streaming and batch data. The basic abstractions of such programs are data sets and transformations. A data set is a collection of arbitrary data objects and transformations describe the transition of one data set to another one. For example, let $X,Y$ be data sets, then a transformation could be seen as a function $t : X \rightarrow Y$. Example transformations are $map$, where for each input object $x_i \in X$ there is exactly one output object $y_i \in Y$, and $reduce$, where all input objects are aggregated to a single one. Further transformations are well known from relational databases, e.g., $join$, $group-by$, $project$, $union$ and $distinct$. To express application logic, transformations are parameterized with user-defined functions. A Flink program may include multiple chained transformations. When executed, Flink handles program optimization as well as data distribution and parallel execution across a cluster of machines.

### 3.2 Graph Representation

We use three object types to represent EPGM data model elements: $graph\ head$, $vertex$ and $edge$. A $graph\ head$ represents the data associated with a single logical graph. Vertices and edges also carry associated data, but additionally need to manage their graph membership as they may be a subset of the logical graph. In consequence, model-specific operators can be triggered explicitly, for example, by writing the result to a file or a database. As none of our operator implementations includes such triggers, multiple operators can be chained and executed as a single Flink program. We show the general idea of mapping graph operations to data set transformations for two of our operators. Listing 1 shows

Listing 1: Implementation of Exclusion on Flink

```java
1: DataSet<id> graphId = secondGraph.getGraphHead();
2: .map(ID_ONLY);
3: DataSet<Vertex> outV = firstGraph.getVertices();
4: .filter(NOT_IN_GRAPH_FILTER)
5: .withBroadcastSet(graphId);
6: DataSet<Edge> outE = firstGraph.getEdges();
7: .filter(NOT_IN_GRAPH_FILTER)
8: .withBroadcastSet(graphId);
9: .join(outV).where(SOURCE_ID).equalTo(VERTEX_ID)
10: .with(KEEP_LEFT_SIDE);
11: .join(outE).where(TARGET_ID).equalTo(VERTEX_ID)
12: .with(KEEP_LEFT_SIDE);
13: return new LogicalGraph(outV, outE);
```

Listing 2: Implementation of Selection on Flink

```java
1: DataSet<GraphHead> outGHeads = coll.getGraphHeads();
2: .filter(predicateFunction);
3: DataSet<id> graphIds = outGHeads.map(ID_ONLY);
4: DataSet<Vertex> outV = coll.getVertices();
5: .filter(IN_ANY_GRAPH_FILTER)
6: .withBroadcastSet(graphIds);
7: DataSet<Edge> outE = coll.getEdges();
8: .filter(IN_ANY_GRAPH_FILTER)
9: .withBroadcastSet(graphIds);
10: return new GraphCollection(outGHeads, outV, outE);
```

identifiers to be generated independently in a distributed environment. Thus, we implemented identifiers using a 128-bit *universally unique identifier*\(^8\). Furthermore, each element has a label of type string and a set of properties. Since EPGM elements are self-descriptive, properties are represented by a key-value map, where the property key is of type string and the property value is encoded in a byte array. Our current implementation supports values of all primitive Java types. Vertices and edges maintain their graph membership in a dedicated set of graph identifiers (graphIds), edges additionally store the identifiers of their incident vertices.

To represent a graph collection, we use a dedicated Flink data set for each element type. In our implementation, a logical graph is a special case of a graph collection where the graph head data set contains a single object. The runtime representation of graph $G_0$ in Figure 1 can be sketched as follows:

- **LogicalGraph** $g_0 =$
  - **DataSet<GraphHead>** graphHead = {
    - <0,'Community', {'interest': 'Databases',...}>}
  - **DataSet<Vertex>** vertices = {
    - <0,'Person', {'name': 'Alice',...},(0,2)>,
    - <1,'Person', {'name': 'Bob',...},(0,2)>
  }
  - **DataSet<Edge>** edges = {
    - <0,'knows',0,1,{'since':'2014'},(0,2)>,
    - <1,'knows',0,1,{'since':2014},(0,2)>
  }

One can see, indicated by the respective graph identifiers, that associated vertices and edges are shared with logical graph $G_2$.

Since we use Flink data sets for graph representation, a graph analytical program is not limited to EPGM operators, but can also benefit from all libraries offered by Flink (e.g., for relational operations, machine learning or graph processing).

### 3.3 Operators

We implemented the GrALa domain specific language using the Java programming language. Each EPGM operator is mapped to a sequence of Flink transformations on the respective data sets. In Flink, program execution needs to be triggered explicitly, for example, by writing the result to a file or a database. As none of our operator implementations includes such triggers, multiple operators can be chained and executed as a single Flink program. We show the general idea of mapping graph operations to data set transformations for two of our operators. Listing 1 shows

\(^8\)docs.oracle.com/javase/7/docs/api/java/util/UUID.html
4. PRELIMINARY EXPERIMENTS

We evaluate our EPGM implementation on a cluster with 16 worker nodes. Each worker consists of an E5-2430 6(12) 2.5 Ghz CPU, 45GB RAM, two 4TB SATA disks and runs openSUSE 13.2. The nodes are connected via 1 Gigabit Ethernet. Our evaluation is based on Hadoop 2.6.0 and Flink 1.0-SNAPSHOT (commit: adbeec2). We run Apache Flink with 12 threads and 40GB memory per worker.

We perform our experimental studies using datasets generated by the Graphalytics benchmark for graph processing platforms. The generator creates heterogeneous social network graphs that have a fixed schema similar to our example in Figure 1 and mimic structural characteristics of real-world networks [4]. Table 2 shows the datasets used throughout the benchmark. The scale factor (SF) denotes the increase of edges of type knows. Since our benchmark primarily involves vertices of type Person and edges of type knows, we added the particular ratio to the table.

The graph analytical program used for benchmarking is presented in Listing 3. The input is the entire social network as a single logical graph. First, we extract the subgraph containing only persons and their mutual relationships. The resulting graph is then transformed to a representation which is limited to information necessary for subsequent operators. Note, that we rename the vertex property birthday to k (line 11). That property key is then used in line 13 as an argument for a specific community detection algorithm [16], which is already implemented in Flink Gelly. The algorithm propagates the property value associated with k through the graph in four iterations. The result is a graph collection containing all found communities. In line 15, we apply the aggregate operator on each of these communities to compute their vertex counts. Then, we use the selection operator to filter communities whose vertex count exceeds a given threshold (see Table 2, Thresh.). The filtered communities are then combined to a single logical graph by applying the reduce operator on the filtered collection. We further group the combined graph by the vertex properties city and gender to see the relations between those groups. Edges are grouped along their incident vertices. By applying groupwise counting, we can find out how many vertices and edges are represented by their respective super entities. In lines 24 and 25, we use aggregation to compute how many super entities are contained in the resulting logical graph. The source code for our benchmark program is available online.9

In Figure 3, we show the results of our first experiments to evaluate the scalability of our implementation. For each con-

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9https://git.io/vgozj
In the first experiment, we evaluated our implementation with respect to growing graph size at a fixed number of workers. In Figure 3(a), one can see that execution time scales nearly linearly from GA.10 to GA.10K. Due to a fixed job initialization time, smaller graphs do not further reduce execution times.

In the second experiment, we evaluated execution time for a fixed graph size (GA.100) but increasing the number of workers. In Figure 3(b), one can see execution times ranging from 1,057 seconds on a single worker to 104 seconds on 16 workers. Figure 3(c) shows the speedup for the same experiment and reveals a nearly linear speedup for up to 8 workers and a slight speedup decrease for 16 workers.

Generally, the largest share of execution time can be traced back to communication between workers. This is mainly caused by join and group-by transformations in our operator implementations. Here, workers need to exchange data among each other which leads to increased network traffic. However, transformations like map and filter do not require communication and can be executed independently by single workers. To reduce data exchange between workers, Flinks optimizer reorders and chains multiple transformations whenever possible.

5. RELATED WORK

Surprisingly, the support for graph collections and associated operators in graph data models has not found much attention, yet. In [10], the authors introduce a formal graph algebra supporting graph collections with heterogeneous attributes on vertices, edges and graphs. Their focus is on graph pattern matching and the construction of new graphs from embeddings. However, graphs are no subgraphs from shared vertex and edge sets but independent graphs that need to be connected explicitly. In [6], vertices and edges are added to so-called domains, which can be seen equivalent to logical graphs. However, there is no support for attributes and advanced operators on domains. In contrast to both approaches, the EPGM is primarily application-driven as we avoid redundancy and additionally offer analytical operators that go beyond pattern matching. A detailed discussion on related work concerning RDF can be found in our technical report on Gradoop [12].

6. CONCLUSIONS AND FUTURE WORK

We presented the EPGM, a graph data model supporting declarative, combinable operators on single graphs and graph collections, including a scalable implementation. Our model suits various applications as it enables the definition of analytical programs on heterogeneous, schema-free graphs. Our first experiments show that we benefit from the underlying framework and its approach to distributed computing. The implementation is open-source, functioning and can be extended to new use cases. Our future work will focus on graph pattern matching and general improvements through graph partitioning, program optimization and reduced memory consumption.

7. ACKNOWLEDGMENTS

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8. REFERENCES